NOTE

Some Novel Expressions for the Propagation Constant of a Uniform Line

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IN a modern communication system the leakance is generally quite small and can be neglected in making propagation computations.

Hence the case is taken of a circuit having resistance, inductance and capacitance.

Let R, L and C be the resistance, inductance and capacitance per mile of the circuit.

Now the formulae which have been developed for the real and imaginary parts of P, the propagation constant, are:

$$P = \alpha + j\beta$$

where

$$\alpha = \sqrt{\tfrac{1}{2}\omega C\{\sqrt{R^2 + \omega^2 L^2} - \omega L\}} \tag{1}$$

$$\beta = \sqrt{\tfrac{1}{2}\omega C\{\sqrt{R^2 + \omega^2 L^2} + \omega L\}} \tag{2}$$

Inspection of these formulae does not reveal any obvious reason for the particular structure of the expressions; however, they may be changed so as to present themselves in a different form.

P can be written:

$$P = \beta(\alpha/\beta + j) \tag{3}$$

Now

$$\frac{\alpha}{\beta} = \sqrt{\frac{\sqrt{R^2 + \omega^2 L^2} - \omega L}{\sqrt{R^2 + \omega^2 L^2} + \omega L}} = \tan\theta \tag{4}$$

where θ is the absolute value of the angle of the characteristic impedance.

Hence

$$P = \beta(\tan\theta + j) \tag{5}$$

The expression in (4) for $\tan\theta$ may be written:

$$\tan\theta = \sqrt{1 - LCV^2} \tag{6}$$

where V = the phase velocity.

Now $LC = 1/V_o^2$

where V_o = the transient velocity.

Hence

$$\tan\theta = \sqrt{1 - V^2/V_o^2} \tag{7}$$

and

$$P = \beta(\sqrt{1 - V^2/V_o^2} + j) \tag{8}$$

Now it can also be shown that for a progressive wave:

$$\sqrt{1 - \frac{V^2}{V_o^2}} = \sqrt{\frac{\frac{1}{2}CE^2 - \frac{1}{2}LI^2}{\frac{1}{2}CE^2 + \frac{1}{2}LI^2}} \tag{9}$$

So that the real part of the propagation constant is equal to the square root of the difference between the electrostatic energy per mile and the electromagnetic energy per mile divided by the sum of the energies (multiplied by β).

This gives the attenuation constant in terms of the most fundamental entity that we know, namely energy.

Also multiplying each side of equation (8) by x the geographical length of the line between the origin and the point under consideration:

$$Px = \beta(\sqrt{1 - V^2/V_o^2}\,x + jx) \tag{10}$$

Now βx is the number of radians (i.e. total phase shift along the length x) and using relativity conceptions,

$$x\sqrt{1 - \frac{V^2}{V_o^2}}$$

is the apparent length of the wave structure in the length x (the waves moving at velocity V) as observed from a fixed point with signals moving at velocity V_o.

Also $\beta x\sqrt{1 - V^2/V_o^2}$ is the number of radians in the apparent length $x\sqrt{1 - V^2/V_o^2}$

The characteristic impedance may be stated in terms of the phase velocity as follows:

$$Z_0 = \frac{1}{CV} - j\frac{RV}{2\omega} \tag{11}$$